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Stable non-supersymmetric supergravity solutions from deformations of the Maldacena-Nuñez background

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Abstract: We study a deformation of the type IIB Maldacena-Nuñez background which arises as the near-horizon limit of NS5 branes wrapped on a two-cycle. This background is dual to a "little string theory" compactified on a two-sphere, a theory which at low energies includes four-dimensional $\mathcal{N} = 1$ super Yang-Mills theory. The deformation we study corresponds to a mass term for some of the scalar fields in this theory, and it breaks supersymmetry completely. In the language of seven-dimensional SO(4) gauged supergravity the deformation involves (at leading order) giving a VEV, depending only on the radial coordinate, to a particular scalar field. We explicitly construct the corresponding solution at leading order in the deformation, both in seven-dimensional and in ten-dimensional supergravity, and we verify that it completely breaks supersymmetry. Since the original background had a mass gap and we are performing a small deformation, the deformed background is guaranteed to be stable even though it is not supersymmetric.

Keywords: Superstring Vacua, AdS-CFT and dS-CFT Correspondence, Supersymmetry Breaking, Supergravity Models.

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Contents

1. Introduction

1.1 The quest for a non-supersymmetric stable background

The principle of holography states that any theory of gravity in d dimensions is equivalent to a non-gravitational theory in $d-1$ dimensions. This principle is completely independent of supersymmetry, but its first explicit realizations in the AdS/CFT correspondence [1]–[4] were all supersymmetric. Attempts to find non-supersymmetric holographic dual pairs starting from the gravitational side, for instance by looking at non-supersymmetric solutions involving an AdS space, encountered the same problems as attempts to find stable non-supersymmetric string theory backgrounds (see e.g. [5] for a discussion). One problem is that loop effects generally generate a potential for the dilaton and any other moduli scalars, which tends to destabilize the vacuum. Another problem is that such backgrounds generically involve tachyonic fields which do not satisfy the Breitenlohner-Freedman (BF) bound, and these also tend to destabilize such spaces. Both types of problems generally show up in the particular example of non-supersymmetric orbifolds [6].

There are various ways to get around these problems and find non-supersymmetric holographic dual pairs. One possibility is to find non-supersymmetric backgrounds which do not have uncharged moduli or tachyons below the BF bound. Such backgrounds do not occur as perturbative string theory solutions, since in string theory the dilaton is always a modulus, but there are many such backgrounds in M theory; for instance, supersymmetric M theory backgrounds of the form $AdS_4 \times M^7$ (where M^7 is a compact space which is not S^7) always have non-supersymmetric twins related to them by "skew-whiffing" [7], and other stable examples also exist. The main problem with such M-theoretic examples is that the corresponding field theories are poorly understood, and it is not known how to get theories resembling QCD in this way.

Another possibility is to start from a supersymmetric background which is dual to a known field theory, and to deform the field theory in a way which breaks supersymmetry; for instance, one can start from a superconformal theory and deform it by a relevant deformation. The simplest example involves starting from $\mathcal{N} = 4$ super Yang-Mills (SYM) theory and adding a mass term for the scalars. The problem with this example (and with other examples of this type) is that giving an equal mass to all the scalars corresponds to deforming by a non-chiral operator which has a large anomalous dimension and is not relevant in the supergravity regime, while deformations which are relevant always involve giving a negative mass-squared to at least one scalar, so they do not lead to a stable non-supersymmetric vacuum. Generally, when one is deforming a theory with a moduli space, one must make sure that the deformation does not generate a potential on the moduli space which would destabilize the vacuum. Since the theories which have supergravity duals are usually strongly coupled, it is difficult to analyze this question. However, several examples are known of deformations which do lead to stable non-supersymmetric backgrounds, starting from [8] where the $\mathcal{N} = 4$ theory was deformed by fermion mass terms. For some values of the masses this deformation breaks supersymmetry and still leads to a stable vacuum [8, 9, 10]. This example (and similar ones) involves fivebranes, so it cannot be fully described by supergravity. Another attempt to break supersymmetry using exactly marginal deformations of two dimensional CFTs was described in [11]; in this case the deformation leads to a non-local theory on the worldsheet, so again one does not get a duality between a standard perturbative string theory and a known field theory. Examples of meta-stable non-supersymmetric backgrounds were recently discussed in [12]. Another way to break supersymmetry is by adding a finite temperature in the field theory, following [13], but here we focus only on Lorentz-invariant configurations.

In this paper we construct the first example of a stable non-supersymmetric supergravity background which is holographically dual to a field theory including four-dimensional Yang-Mills (YM) theory.[1] The field theory in this case is a deformation of "little string

[1]More precisely, this is the first example (as far as we know) which is the low-energy limit of a string theory which can be chosen to be weakly coupled everywhere; an example involving strong coupling, coming from compactified D4 branes, was described in [13].

theory" (see [14] for a review) compactified on a two-sphere, whose holographic dual was found in [15]. This theory arises for instance from a decoupling limit of N NS5 branes wrapped on a two-cycle in a Calabi-Yau manifold. At low energies (in the case of type IIB NS5 branes) this theory includes four-dimensional YM theory with gauge group SU(N), though in the supergravity approximation these modes are inseparable from other modes coming from the compactification [15]. As found in [15], this background has a mass gap, so we are guaranteed that it will be stable under small deformations, even if they break supersymmetry. In this paper we will describe in detail a particular deformation of this theory, corresponding to a six-dimensional mass term for scalar fields. The particular example we analyze is not in the same universality class as pure YM theory because it includes a massless adjoint fermion with a U(1) R-symmetry (classically in the UV) which protects it from acquiring a mass; however, it should be possible also to construct generalizations of our example which could be in the same universality class as pure YM. In our case, as in other holographic constructions, going to a limit corresponding to a string theory for pure YM theory or QCD (which is our eventual goal), without any additional fields, requires going beyond the supergravity approximation and performing a full string theory analysis, which is beyond our current capabilities.

1.2 The Maldacena-Nuñez supersymmetric background

A stack of N flat NS5 branes in type IIB string theory gives rise to a linear dilaton background, which is dual, in an appropriate decoupling limit, to a "little string theory" [16]. At low energies this theory includes a six-dimensional $\mathcal{N} = (1,1)$ SYM theory with SU(N) gauge group, which includes four adjoint scalars Φ_a. Maldacena and Nuñez [15] analyzed the supergravity solution corresponding to wrapping the fivebranes on a two-sphere S^2, with the remaining four directions spanning a four-dimensional Minkowski space. By twisting the normal bundle appropriately (this is automatic when the fivebranes wrap a two-cycle in a Calabi-Yau manifold) one fourth of the original supersymmetry is maintained, corresponding to four-dimensional $\mathcal{N} = 1$ supersymmetry, and the scalars Φ_a all become massive.[2] At low energies this theory includes the four-dimensional $\mathcal{N} = 1$ pure SYM theory. The two-sphere which the theory is compactified on appears also in the dual supergravity background, and its radius decreases as we go from the UV (large radial coordinate) to the IR (small radial coordinate).

All the supergravity fields which are non-zero in the solution are contained in the ten-dimensional type I supergravity sector of the type IIB theory. For fivebranes in flat space, the isometries of the transverse three-sphere give rise to an $SO(4) \equiv SU(2)_L \times SU(2)_R$ symmetry which is the R-symmetry of the corresponding theory. In the theory of the wrapped fivebranes, the remaining R-symmetry is just a U(1) subgroup of $SU(2)_L$. Reducing type I supergravity on the three-sphere gives rise to a seven-dimensional SO(4) gauged supergravity [19], which is a consistent truncation of the full theory. The four-dimensional Minkowski space is merely a spectator in the solution, so the relevant gauged supergravity can be thought of as three-dimensional, or four-dimensional if we keep the time direction.

[2]Analogous solutions in which the compactification maintains half of the supersymmetry were analyzed in [17, 18].

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In the seven dimensional supergravity description, the twisting of the normal bundle is achieved by turning on one of the $\mathrm{SU}(2)_L$ gauge fields, which is taken to be A_L^3. For the solution of [15] it is sufficient to work with a supergravity which is further truncated, and includes only the $\mathrm{SU}(2)_L$ gauge fields, since no fields charged under $\mathrm{SU}(2)_R$ participate in the solution. The naive solution in which one turns on only A_L^3 is analogous to an SU(2) Dirac monopole, and is unphysically singular. However, one can find a smooth solution with the same asymptotic behaviour by turning on also the A_L^1, A_L^2 gauge fields. This was carried out in the four-dimensional SU(2) gauged supergravity context by Chamseddine and Volkov [20, 21]. Maldacena and Nuñez translated that solution to the context of seven-dimensional supergravity, and raised the solution to ten dimensions using the results of [22, 23], to get a smooth solution describing the fivebranes compactified on S^2.

The singular solution exhibits a classical U(1) R-symmetry which is broken by anomalies to $\mathbb{Z}_{2N}$. The instantons responsible for this are given, roughly, by fundamental string worldsheets wrapped on the two-sphere [15]. The smoothed solution further breaks the symmetry spontaneously to $\mathbb{Z}_2$, as expected from the field theory point of view. Maldacena and Nuñez also S-dualized the solution to represent wrapped D5 branes, and found the behaviour of the string tension, glueball masses and domain wall tensions of the SYM theory. As usual in such cases, when the supergravity approximation is valid, the typical mass of the lightest glueballs, Λ_{QCD}, is of the same order as the masses of the Kaluza-Klein modes on the two-sphere, and the SYM theory is not really decoupled from those additional degrees of freedom.

1.3 The supersymmetry breaking perturbation

In this paper we explore the supergravity solution which is dual to explicitly breaking the supersymmetry in the "little string theory" (LST). At low energies, the $\mathcal{N} = (1,1)$ LST coming from N type IIB NS5 branes reduces to the six dimensional $\mathcal{N} = (1,1)$ $\mathrm{SU}(N)$ SYM theory, which is the dimensional reduction of ten-dimensional SYM to six dimensions. This theory includes four adjoint scalar fields Φ_a in the $(\mathbf{2},\mathbf{2})$ representation of the global SO(4) R-symmetry. The simplest chiral operator in the theory, as in other maximally supersymmetric SYM theories, is $\mathcal{O} = X^{ab}\,\mathrm{tr}\,\Phi_a\Phi_b$ where X is a traceless symmetric SO(4) matrix; this operator is in the $(\mathbf{3},\mathbf{3})$ representation of SO(4). In appendix A we define a basis X_{lr} for this representation, where the indices l, r are in the adjoint of $\mathrm{SU}(2)_{L,R}$, respectively. We wrote the operator $\mathcal{O}$ as an operator of the low-energy SYM theory, but it is actually a chiral operator in the full LST [16], which reduces to this form at low energies. Our deformation will involve adding a term of the form $\epsilon\mathcal{O}$ to the lagrangian of the six-dimensional theory, and we will find the dual background at leading order in perturbation theory in ϵ (the dimensions will be set by the string tension, which is also the inverse Yang-Mills coupling in the six-dimensional theory, or by the radius of the two-sphere on which the theory is compactified).

In the six-dimensional theory we can always diagonalize the matrix X by a global SO(4) transformation. A deformation by $\mathcal{O}$ is thus determined by three invariants, or alternatively by four eigenvalues whose sum is zero. In the compactified theory the $\mathrm{SU}(2)_L$ is broken to a U(1) which is the R-symmetry group (in our conventions, this U(1) corresponds to the adjoint index $l = 3$ of $\mathrm{SU}(2)_L$). Clearly, there is a difference between choosing the deformation to be charged or uncharged under this U(1). In this paper we will analyze the uncharged case. In this case the matrix X appearing in the deformation is of the form $v^r X_{3r}$ (in the conventions of appendix A) for some arbitrary vector v^r, and its eigenvalues are proportional to $\{-1, -1, +1, +1\}$. Without loss of generality, we may take $X \equiv X_{33}$ and the corresponding operator is $\mathcal{O} = \mathrm{tr}\left(\Phi_1^2 - \Phi_2^2 - \Phi_3^2 + \Phi_4^2\right)$. As our deformation will preserve the classical U(1) R-symmetry in the UV, its breaking will remain as in [15]. In particular, this symmetry forbids (before it is spontaneously broken) the generation of a mass term for the gluino in the four-dimensional $\mathcal{N} = 1$ SYM multiplet, so even after the deformation we will have (classically) a massless four-dimensional adjoint fermion, despite the absence of supersymmetry.

From the point of view of the six-dimensional theory we are giving some of the scalars in the SYM theory a negative mass squared, making them tachyonic. Thus, if we try to perform this SUSY-breaking deformation directly in the six-dimensional theory, it will destabilize the vacuum and the theory will run to large values of Φ_1 and Φ_4 (if $\epsilon > 0$). However, when we compactify on S^2 to four dimensions, all the modes of the fields Φ_a become massive, with a mass of at least the order of the inverse compactification scale. Thus, as long as ϵ is small enough, the deformation in the compactified theory does not destabilize the vacuum, but just changes the masses of the already massive fields. Another way to see that the resulting theory must be stable (at least for small values of $|\epsilon|$) is to note that the theory before the deformation had a mass gap, so it cannot be destabilized by any small deformation.

Since the fermion masses get no contribution at leading order in ϵ, supersymmetry is explicitly broken by the deformation. The fields in the dual background which are dual to the operator $\mathcal{O}$ were described in [16] for the uncompactified six dimensional theory; they involve a squashing of the metric on the 3-sphere. In the seven-dimensional supergravity theory they are nine scalar fields c_{lr}. Since the operator $\mathcal{O}$ is charged under both $\mathrm{SU}(2)_L$ and $\mathrm{SU}(2)_R$, we can no longer work (as in [15]) with the truncation of the seven-dimensional supergravity to $\mathrm{SU}(2)_L$, but have to deal with the full SO(4) gauged supergravity. Since supersymmetry is broken, we can't use the BPS first order differential equations to find the new solution, and we must deal with the second order equations of motion.

The organization of this paper is as follows. In the next section 2 we translate the SO(4) gauged supergravity lagrangian of [22] to the seven-dimensional $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ language, including the nine scalars and working to quadratic order in ϵ. Section 3 describes the singular four-dimensional monopole solution and the smooth monopole solution of Chamseddine and Volkov, and studies their symmetries. In section 4 we derive the order ϵ equations of motion for the perturbation around the Chamseddine-Volkov solution, and obtain the constraints imposed by respecting the symmetries. In section 5 we specialize to the most symmetric case, where the scalar depends only upon the radial direction ρ, and find the solutions for the singular and the non-singular cases. In section 6 the solution is raised to ten dimensions according to [22]. In section 7 we S-dualize the solution and look

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at the string tension. In section 8 we explicitly show that our ten-dimensional supergravity background breaks supersymmetry. We conclude with a summary and discussion. Some matrix conventions and details of the calculations are relegated to three appendices A, B and C.

2. The supergravity lagrangian

The authors of [22] study consistent reductions of a D dimensional theory, containing the metric, a dilaton and a Kalb-Ramond field, down to $D-3$ dimensions, by the Kaluza-Klein mechanism of compactification on an S^3. We are, of course, interested in the case of $D = 10$ supergravity, giving rise to seven-dimensional, SO(4) gauged supergravity. The massless fields in this theory are the metric, a scalar Y, the SO(4) gauge fields $A^{ab}_{(1)}$, a symmetric unimodular matrix $\tilde{T}_{ab}$ (that is $\tilde{T}_{ab} = \tilde{T}_{ba}$ and $\det \tilde{T} = 1$), and also a two-form potential $A_{(2)}$. We will denote the SO(4) gauge coupling constant by $\hat{g}$.

The indices a,b,c,d,$\ldots$ $= 1, 2, 3, 4$ are SO(4) vector indices. $A_{(1)}$ is in the adjoint representation, **6**, of SO(4),

$$A^{ab}_{(1)} = -A^{ba}_{(1)}\,, \tag{2.1}$$

while $\tilde{T}$ is in the representation **9**. The SO(4) decomposes to $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ where the left (right) subgroups have (anti) self-dual gauge potentials:

$$A^{ab}_{(1)} = \pm\frac{1}{2}\epsilon_{abcd}A^{cd}_{(1)}\,. \tag{2.2}$$

We will denote the left and right SU(2) gauge potentials as A^l_L and A^r_R respectively, where l,m,n,$\ldots$ $= 1, 2, 3$ are $\mathrm{SU}(2)_L$ adjoint indices, or equivalently SO(3) vector indices, and similarly r,s,t,$\ldots = 1, 2, 3$ for $\mathrm{SU}(2)_R$.

Under this decomposition, the gauge potential representation obviously decomposes as $\mathbf{6} = (\mathbf{3}, \mathbf{1}) + (\mathbf{1}, \mathbf{3})$, while $\mathbf{9} = (\mathbf{3}, \mathbf{3})$. We choose a certain embedding of the $\mathrm{SU}(2)_L$ and $\mathrm{SU}(2)_R$ generators, α^l_L and α^r_R respectively, in the SO(4) adjoint representation. We also choose a specific representation X_{lr} of the tangent space of the $\tilde{T}$ at the origin $\tilde{T} = \delta_{ab}$ (for each $l = 1, 2, 3$ and $r = 1, 2, 3$, X_{lr} is a 4×4 traceless matrix $(X_{lr})_{ab}$). The details are described in appendix A. We define the fields c_{lr} by the parameterization

$$\tilde{T} = \exp Q\,, \tag{2.3}$$

where

$$Q = c_{lr}X_{lr}\,, \qquad \mathrm{tr}\,Q = 0\,, \tag{2.4}$$

We will also denote by c the corresponding 3×3 matrix.

Let us define the field strengths

$$F^{ab}_{(2)} = dA^{ab}_{(1)} + \hat{g}A^{ac}_{(1)} \wedge A^{cb}_{(1)} \tag{2.5}$$

and

$$F_{(3)} = dA_{(2)} + \frac{1}{8}\epsilon_{abcd}\left(F^{ab}_{(2)} \wedge A^{cd}_{(1)} - \frac{1}{3}\hat{g}A^{ab}_{(1)} \wedge A^{ce}_{(1)} \wedge A^{ed}_{(1)}\right), \tag{2.6}$$

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as well as the covariant derivative of $\tilde{T}_{ab}$,

$$\mathcal{D}\tilde{T}_{ab} = d\tilde{T}_{ab} + \hat{g}(A^{ac}_{(1)}\tilde{T}_{cb} + A^{bc}_{(1)}\tilde{T}_{ac}) \,, \tag{2.7}$$

compatible with its SO(4) index structure.

The Einstein frame lagrangian of the $(D-3)$-dimensional massless theory was found to be [22], in the language of differential forms,

$$\begin{aligned}\mathcal{L}_{D-3} = {} & R \star 1 - \frac{D-5}{16} Y^{-2} \star dY \wedge dY - \frac{1}{4}\tilde{T}^{-1}_{ab} \star \mathcal{D}\tilde{T}_{bc} \wedge \tilde{T}^{-1}_{cd}\mathcal{D}\tilde{T}_{da} - \\ & -\frac{1}{2}Y^{-1} \star F_{(3)} \wedge F_{(3)} \frac{1}{4} Y^{-1/2} - \tilde{T}^{-1}_{ac}\tilde{T}^{-1}_{bd} \star F^{ab}_{(2)} \wedge -F^{cd}_{(2)} V \star 1 \,,\end{aligned} \tag{2.8}$$

where the potential V is given by

$$V = \frac{1}{2}\hat{g}^2 Y^{1/2}\left(2\tilde{T}_{ab}\tilde{T}_{ab} - (\tilde{T}_{aa})^2\right) . \tag{2.9}$$

The first term in the lagrangian is the Einstein-Hilbert term. The second is the kinetic energy of the scalar Y, the third is that of the scalars in the **9** representation, while the fourth is that of $A_{(2)}$. The fifth term is the Maxwell term.

Let us look at the $A_{(2)}$ kinetic term. If we truncate the gauge group to the diagonal $\mathrm{SU}(2)_D \subset \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$, as in [17, 18], the scope of the indices $a,b,c,d,\ldots$ is reduced to (say) $1,2,3$. Thus ϵ_{abcd} vanishes in (2.6), and we are left with $F_{(3)} = dA_{(2)}$. The lagrangian is therefore quadratic in $A_{(2)}$, and $A_{(2)}$ can be decoupled from the rest of the fields. Naively, for the full SO(4) theory this decoupling is not possible. However, in the case $D = 10$ this theory can also be written [23] in an equivalent form [24, 25], using a field $A_{(3)}$ dual to $A_{(2)}$. The lagrangian is given in [23, equation (19)]. We will be interested in the ansatz in which the seven-dimensional space is a product of a four-dimensional Minkowski space and a three-dimensional space, and where the four-dimensional Poincaré invariance is maintained. The latter form of the lagrangian makes it easy to see that in this case, the $A_{(3)}$ field has no sources from the other fields. Therefore, its action is quadratic and it can be decoupled. Thus, we will henceforth ignore this extra field. This has also been done in the analysis of [15], where the gauge group is truncated to $\mathrm{SU}(2)_L$.

We will now write the scalar lagrangian L_7,

$$\mathcal{L}_7 = \sqrt{\det g_{\mu\nu}}\; L_7 \; dx^0 \wedge dx^1 \wedge \cdots \wedge dx^6 \,, \tag{2.10}$$

using the standard definitions

$$\begin{aligned} A^{ab}_{(1)} &= A^{ab}_{\mu} dx^{\mu} \,, \\ F^{ab}_{\mu\nu} &= \partial_{\mu}A^{ab}_{\nu} - \partial_{\nu}A^{ab}_{\mu} + \hat{g}(A^{ac}_{\mu}A^{cb}_{\nu} - A^{ac}_{\nu}A^{cb}_{\mu}) \,, \\ \mathcal{D}_{\mu}\tilde{T}_{ab} &= \partial_{\mu}\tilde{T}_{ab} + \hat{g}(A^{ac}_{\mu}\tilde{T}_{cb} + A^{bc}_{\mu}\tilde{T}_{ac}) \,. \end{aligned} \tag{2.11}$$

We get

$$\begin{aligned} L_7 = {} & R - \frac{5}{16}Y^{-2}\partial_{\mu}Y\partial^{\mu}Y - \frac{1}{4}\tilde{T}^{-1}_{ab}\mathcal{D}_{\mu}\tilde{T}_{bc}\tilde{T}^{-1}_{cd}\mathcal{D}^{\mu}\tilde{T}_{da} - \\ & -\frac{1}{8}Y^{-1/2}\tilde{T}^{-1}_{ac}\tilde{T}^{-1}_{bd}F^{ab}_{\mu\nu}F^{cd\mu\nu} - V \,. \end{aligned} \tag{2.12}$$

Notice the numerical coefficient 1/8 in the Maxwell term. This is, however, the canonical normalization, in light of (2.1). When truncating to $\mathrm{SU}(2)_D$, by limiting the scope of the indices $a, b, \ldots$, this normalization is kept. However, when decomposing SO(4) as $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$, or when truncating to $\mathrm{SU}(2)_L$, the normalization is non-standard. Thus the gauge coupling constant g of the SU(2) groups will differ from the SO(4) gauge coupling constant $\hat{g}$.

We define the $\mathrm{SU}(2)_L$ and $\mathrm{SU}(2)_R$ gauge fields, A_L^l and A_R^r respectively, by

$$-iA = \sqrt{2}(A_L^l \alpha_L^l + A_R^r \alpha_R^r)\,, \tag{2.13}$$

where A is viewed as a 4×4 matrix in the indices a, b, besides the spatial index μ. Viewing likewise F as a 4×4 matrix, we get

$$-iF = \sqrt{2}(F_L^l \alpha_L^l + F_R^r \alpha_R^r)\,, \tag{2.14}$$

with

$$\begin{aligned} F_{L\mu\nu}^l &= \partial_\mu A_{L\nu}^l - \partial_\nu A_{L\mu}^l - \sqrt{2}\hat{g}\epsilon_{lmn} A_{L\mu}^m A_{L\nu}^n\,, \\ F_{R\mu\nu}^r &= \partial_\mu A_{R\nu}^r - \partial_\nu A_{R\mu}^r - \sqrt{2}\hat{g}\epsilon_{rst} A_{R\mu}^s A_{R\nu}^t\,. \end{aligned} \tag{2.15}$$

Apart from reverting to the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ language, we also wish to expand the lagrangian near the origin $\tilde{T}_{ab} = \delta_{ab}$, namely, to expand in powers of c. The field c is dual to the operator $\mathcal{O}$ in LST, and when we deform the LST lagrangian by $\epsilon\mathcal{O}$, c will be proportional to ϵ, which we take to be small. In order to solve the equations of motion up to first order in ϵ, it suffices to keep terms in the lagrangian up to second order in c. In appendix B we calculate the various terms of the lagrangian (2.12) to second order in the fields c_{lr} and their derivatives. We will also substitute

$$Y = e^y\,, \tag{2.16}$$

where y is proportional to the dilaton.

The final form of the lagrangian, gathering all the terms and defining

$$g \equiv -\sqrt{2}\hat{g} \tag{2.17}$$

in order to get the standard coupling, is

$$\begin{aligned} L_7 = {} & R - \frac{5}{16}\partial_\mu y \partial^\mu y - \mathcal{D}_\mu c_{lr} \mathcal{D}^\mu c_{lr} - \\ & -\frac{1}{4}e^{-y/2}(F_L^l F_L^l + F_R^r F_R^r + 4F_L^l F_R^r c_{lr} + 2(F_L^l F_L^m c_{lr} c_{mr} + F_R^r F_R^s c_{lr} c_{ls})) + \\ & +2g^2 e^{y/2} + O(c^3)\,, \end{aligned} \tag{2.18}$$

where

$$\begin{aligned} F_{L\mu\nu}^l &= \partial_\mu A_{L\nu}^l - \partial_\nu A_{L\mu}^l + g\epsilon_{lmn} A_{L\mu}^m A_{L\nu}^n\,, \\ F_{R\mu\nu}^r &= \partial_\mu A_{R\nu}^r - \partial_\nu A_{R\mu}^r + g\epsilon_{rst} A_{R\mu}^s A_{R\nu}^t\,, \\ \mathcal{D}_\mu c_{lr} &= \partial_\mu c_{lr} + g(\epsilon_{lmn} A_{L\mu}^m c_{nr} + \epsilon_{rst} A_{R\mu}^s c_{lt})\,, \end{aligned} \tag{2.19}$$

and where, for example, $F_L^l F_L^l \equiv F_L^{l\mu\nu} F_{L\mu\nu}^l$.

3. The Chamseddine-Volkov solution

The lagrangian (2.18) can be truncated to an $\mathrm{SU}(2)_L$ gauged supergravity theory. Indeed, it is an $\mathrm{SU}(2)_R$ invariant, so all the fields charged under $\mathrm{SU}(2)_R$ appear at least quadratically in all the terms, and can be decoupled. The resulting lagrangian after decoupling the fields A^r_R and c_{lr} is

$$L_7 = R - \frac{5}{16}\partial_\mu y \partial^\mu y - \frac{1}{4}e^{-y/2} F^l_L F^l_L + 2g^2 e^{y/2} \,. \tag{3.1}$$

We will look for warped geometry solutions having a four-dimensional Minkowski space factor (with coordinates x_μ). Those solutions are therefore essentially three-dimensional, and were investigated in the context of four-dimensional supergravity [20, 21]. Moreover, the remaining three-dimensional geometry will have an SO(3) symmetry, and will be a warped product of a two-sphere (with coordinates θ, ϕ) and a half line (the radial coordinate ρ). The general form of the geometry in the string frame, modulu the reparameterization invariance of ρ, corresponding to N wrapped NS5 branes [15] is

$$ds^2_{7,\mathrm{st}} = dx_\mu dx^\mu + N\left(d\rho^2 + e^{2h(\rho)}(d\theta^2 + \sin^2\theta d\phi^2)\right) . \tag{3.2}$$

We will work in the Einstein frame, whose metric is given by

$$ds^2_7 = e^{-y(\rho)/2}\, ds^2_{7,\mathrm{st}} \,. \tag{3.3}$$

By looking at the radius of the transverse S^3 we find that N is related to the coupling constant g of the gauged supergravity by

$$N = \frac{2}{g^2} \,. \tag{3.4}$$

There is a solution involving only the gauge field A^3_L (the lagrangian can be further truncated to the theory containing only that $\mathrm{U}(1) \subset \mathrm{SU}(2)_L$). It reads

$$e^{2h(\rho)} = \rho \,, \tag{3.5}$$

$$y(\rho) = \frac{8}{5}\left(\varphi_0 - \rho + \frac{1}{4}\log\rho\right) , \tag{3.6}$$

$$A^3_L = \frac{1}{g}\cos\theta d\phi \,, \tag{3.7}$$

(that is, $A^3_{L\phi} = (\cos\theta)/g$ and all its other components vanish). This solution, however, is singular at $\rho = 0$, and thus it is unphysical.

Let us examine the symmetries of this solution. The field strength is

$$F^3_L = -\frac{1}{g}\omega \,, \tag{3.8}$$

where

$$\omega = \sin\theta\, d\theta \wedge d\phi \tag{3.9}$$

is the SO(3) invariant volume form of S^2. This enables the whole solution to be SO(3) invariant, and in particular the scalars y and h to be radial functions only. The symmetry under an infinitesimal rotation by $\hat{\epsilon}$ around the z axis,

$$\begin{aligned} \delta_3\theta &= 0\,, \\ \delta_3\phi &= \hat{\epsilon}\,, \end{aligned} \tag{3.10}$$

is obvious, as A^3_L depends on ϕ only through $d\phi$ which is invariant. Let us now look at a rotation around the y axis,

$$\begin{aligned} \delta_2\theta &= \hat{\epsilon}\cos\phi\,, \\ \delta_2\phi &= -\hat{\epsilon}\cot\theta\sin\phi\,. \end{aligned} \tag{3.11}$$

This time A^3_L, given by (3.7), is not invariant, but the change is a gauge transformation,

$$\delta_2 A^3_L = \frac{1}{g}\,d\Lambda\,, \tag{3.12}$$

with

$$\Lambda = -\hat{\epsilon}\,\frac{\sin\phi}{\sin\theta}\,. \tag{3.13}$$

Of course, δ_2 and δ_3 generate the whole SO(3) symmetry.

The singular solution can be smoothed in the full $\mathrm{SU}(2)_L$ gauged supergravity theory [20, 21]. We choose the ansatz

$$A_L = \frac{1}{g}\left(a(\rho)\,d\theta\,\tau^1 + a(\rho)\,\sin\theta\,d\phi\,\tau^2 + \cos\theta\,d\phi\,\tau^3\right), \tag{3.14}$$

where $\tau^l = \sigma^l/2$ are the SU(2) generators. Obviously, δ_3 is still a symmetry, and it can be verified that δ_2 is still equivalent to the same singular gauge transformation, which is now manifestly non-abelian,

$$\delta_2 A_L = \frac{1}{g}\,d\Lambda + i\,[\Lambda, A_L]\,, \tag{3.15}$$

with

$$\Lambda = -\hat{\epsilon}\,\frac{\sin\phi}{\sin\theta}\,\tau^3\,. \tag{3.16}$$

The appropriate supersymmetric solution is [20, 21]

$$\begin{aligned} e^{2h(\rho)} &= \rho\coth(2\rho) - \frac{\rho^2}{\sinh^2(2\rho)} - \frac{1}{4}\,, && (3.17) \\ y(\rho) &= \frac{8}{5}\left(\varphi_0 + \frac{1}{2}\log\left(\frac{2e^{h(\rho)}}{\sinh(2\rho)}\right)\right), && (3.18) \\ a(\rho) &= \frac{2\rho}{\sinh(2\rho)}\,. && (3.19) \end{aligned}$$

The non-zero components of the gauge field strength are

$$F^1_{L\rho\theta} = -F^1_{L\theta\rho} = \frac{1}{g}\, a'(\rho)\,, \tag{3.20}$$

$$F^2_{L\rho\phi} = -F^2_{L\phi\rho} = \frac{1}{g}\sin\theta\, a'(\rho)\,, \tag{3.21}$$

$$F^3_{L\theta\phi} = -F^3_{L\phi\theta} = -\frac{1}{g}\sin\theta\left(1 - a^2(\rho)\right). \tag{3.22}$$

This solution approaches the singular one for $\rho \to \infty$. In fact, if in the singular solution we substitute $\rho - 1/4$ instead of ρ, and change φ_0 appropriately, the difference between the solutions involves only exponentially small terms for large ρ.

Most importantly, this solution is regular at $\rho = 0$. In fact, the geometry is becoming flat for $\rho \to 0$, as

$$e^{2h(\rho)} = \rho^2 + O(\rho^4)\,, \tag{3.23}$$

$$y(\rho) = \frac{8}{5}\varphi_0 + O(\rho^2)\,. \tag{3.24}$$

4. The linearized equation for the scalars

The equations of motion of the fields charged under $\mathrm{SU}(2)_R$, that is, the gauge vectors A^r_R and the scalars c_{lr} in the $\mathbf{9} = (\mathbf{3},\mathbf{3})$ representation, derived from the lagrangian (2.18), are

$$\begin{aligned}\mathcal{D}_\mu\mathcal{D}^\mu c_{lr} - \frac{1}{2}e^{-y/2}(F^l_L F^r_R + F^l_L F^m_L c_{mr} + F^r_R F^s_R c_{ls}) + O(c^2) &= 0\,,\\ \mathcal{D}_\mu\left[e^{-y/2}(F^{r\mu\nu}_R + 2F^{l\mu\nu}_L c_{lr} + 2F^{s\mu\nu}_R c_{lr}c_{ls})\right] + O(c^3) &= 0\,,\end{aligned} \tag{4.1}$$

where $\mathcal{D}_\mu$ is a metric covariant derivative as well as a gauge covariant derivative. We wish to solve those equations in perturbation theory around the regular background described in the previous section. We take the perturbation to be proportional to a small parameter ϵ, so that we can use the linearized form of the equations,

$$\mathcal{D}_\mu\mathcal{D}^\mu c_{lr} - \frac{1}{2}e^{-y/2}(F^l_L F^r_R + F^l_L F^m_L c_{mr}) + O(\epsilon^2) = 0\,, \tag{4.2}$$

$$\mathcal{D}_\mu\left[e^{-y/2}(F^{r\mu\nu}_R + 2F^{l\mu\nu}_L c_{lr})\right] + O(\epsilon^2) = 0\,. \tag{4.3}$$

At first order in ϵ, the fields A^r_R can be taken as three independent abelian gauge vectors, with the corresponding F^r_R and covariant derivatives $\mathcal{D}_\mu$. Therefore, the equations above decouple into three sets of identical equations for $r = 1,2,3$, and the general solution is a superposition of three copies of the decoupled solutions. By a global $\mathrm{SU}(2)_R$ transformation we may, if we wish, take $r = 3$ without loss of generality, and we will denote $A_R \equiv A^3_R$, $F_R \equiv F^3_R$ and $c_l \equiv c_{l3}$, with

$$\mathcal{D}_\mu c_l = \partial_\mu c_l + g\epsilon_{lmn}A^m_{L\mu}c_n\,. \tag{4.4}$$

The simplest solutions will be those retaining the symmetries of the background. We begin by looking at the fields c_l. Let us define $c = c_1\tau^1 + c_2\tau^2 + c_3\tau^3$. Demanding the invariance of c under the symmetry δ_3 is obviously equivalent to $\partial c/\partial\phi = 0$, that is, as a function of the angles, c is dependent upon θ only:

$$c(\theta, \phi) = f_3(\theta)\,, \tag{4.5}$$

for an arbitrary function f_3.

Invariance of c under the symmetry δ_2 requires that the geometric transition

$$\delta_2 c = \frac{\partial c}{\partial\theta}\,\delta_2\theta + \frac{\partial c}{\partial\phi}\,\delta_2\phi \tag{4.6}$$

will be equivalent to the gauge transformation (3.13)

$$\delta c = i\,[\Lambda, c] = \hat{\epsilon}\,\frac{\sin\phi}{\sin\theta}(c_1\tau^2 - c_2\tau^1)\,, \tag{4.7}$$

which gives

$$\begin{aligned}
\delta c_1 &= -\hat{\epsilon}\,\frac{\sin\phi}{\sin\theta}c_2\,,\\
\delta c_2 &= +\hat{\epsilon}\,\frac{\sin\phi}{\sin\theta}c_1\,,\\
\delta c_3 &= 0\,.
\end{aligned} \tag{4.8}$$

This amounts to the partial differential equations

$$\begin{aligned}
Dc_1 &= -c_2\,,\\
Dc_2 &= +c_1\,,\\
Dc_3 &= 0\,,
\end{aligned} \tag{4.9}$$

with

$$D \equiv \sin\theta\cot\phi\,\frac{\partial}{\partial\theta} - \cos\theta\,\frac{\partial}{\partial\phi}\,. \tag{4.10}$$

Changing coordinates to

$$\begin{aligned}
u &= \cos\theta\tan\phi\,,\\
v &= \sin\theta\sin\phi\,,
\end{aligned} \tag{4.11}$$

we have

$$D = -(1+u^2)\frac{\partial}{\partial u}\,, \tag{4.12}$$

and the solution can be seen to be

$$c_1 = \frac{1}{\sqrt{1+u^{-2}}}f_2(v)\,, \tag{4.13}$$

$$c_2 = \frac{1}{\sqrt{1+u^{+2}}}f_2(v)\,, \tag{4.14}$$

$$c_3 = \hat{f}_2(v)\,, \tag{4.15}$$

for arbitrary functions $f_2, \hat{f}_2$, or the exchanged solution $c_1 \to c_2, \quad c_2 \to -c_1$.

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5. Solutions of the linearized equations

Demanding invariance under both δ_2 (4.13), (4.14), (4.15) and δ_3 (4.5) is easily seen to lead to the vanishing of c_1 and c_2, and to c_3 being constant as a function of the angles, that is, being only a radial function. We will assume this in the rest of the paper.

In order for the field strength to respect this SO(3) symmetry, it can be seen that the gauge vector must be of the form

$$A_R = \epsilon \cdot \frac{\gamma}{g} \cos\theta \, d\phi \,, \tag{5.1}$$

$$F_R = \epsilon \cdot \left(-\frac{\gamma}{g}\right) \sin\theta \, d\theta \wedge d\phi \,, \tag{5.2}$$

where γ depends only on the radial coordinate. This is nothing else than the form (3.7) of A_L^3 in the singular background. The equation of motion for A_R (4.3) can be seen to be satisfied if and only if γ is a constant. Note that with only c_3 present, the only non-vanishing components of $F_L^{l\mu\nu} c_l$ are for $\mu = \theta, \nu = \phi$ or vice versa.

However, this solution is singular at the origin $\rho = 0$. There, the contribution to the action from the sphere, parameterized by θ and ϕ, at a radial coordinate $\rho = \rho_0$, can be seen from the lagrangian (2.18) and the metric approximation (3.23), (3.24) to behave like

$$\int F_{R\mu\nu} F_R^{\mu\nu} \, \rho_0^2 \sin\theta \, d\theta \, d\phi \sim \rho_0^{-2}, \tag{5.3}$$

and the contribution from $\rho \geq \rho_0$ diverges as ρ_0^{-1} when ρ_0 tends to zero. Therefore, this solution is physically unacceptable, and we will set $A_R = 0$ from here on.

We remain with the non-trivial equation of motion (4.2) for c_l, which explicitly reads

$$\begin{aligned} c_3''(\rho) + \left(-\frac{5}{4} y'(\rho) + 2h'(\rho)\right) c_3'(\rho) + \\ + \left(-2e^{-2h(\rho)} \, a^2(\rho) - \frac{1}{2} \, e^{-4h(\rho)} \left(1 - a^2(\rho)\right)^2\right) c_3(\rho) + O(\epsilon^2) = 0 \,. \end{aligned} \tag{5.4}$$

Let us first of all study this equation in the limits $\rho \to 0$ and $\rho \to \infty$. In the former limit, the space is flat (3.23), (3.24) and the gauge vector (3.14) tends to a constant,

$$A_L(\rho) \approx \frac{1}{g} \left(d\theta \, \tau^1 + \sin\theta \, d\phi \, \tau^2 + \cos\theta \, d\phi \, \tau^3\right), \tag{5.5}$$

since from (3.19) we have

$$a(\rho) = 1 + O(\rho^2). \tag{5.6}$$

Moreover, the field strength (3.20), (3.21), (3.22) vanishes in this limit,

$$F_L \approx 0 \,. \tag{5.7}$$

Equation (5.4) turns in this limit into

$$c_3''(\rho) + \frac{2}{\rho} c_3'(\rho) - \frac{2}{\rho^2} c_3(\rho) + O(\epsilon^2) = 0 \,, \tag{5.8}$$

two of whose independent solutions are

$$
\begin{aligned}
c_3(\rho) &= \epsilon\rho^{-2} + O(\epsilon^2)\,, \\
c_3(\rho) &= \epsilon\rho + O(\epsilon^2)\,.
\end{aligned} \tag{5.9}
$$

A priori, we can take a linear combination of those solutions with arbitrary coefficients. However, we must demand that the coefficient of the first solution vanishes, or else the solution diverges and is singular at the origin. At first glance, the second solution might also seem to be singular at the origin, having a cusp like that of the absolute value function, and therefore being non-differentiable. However, gauge-invariant objects like c_3^2 are smooth, and the gauge covariant object appearing in the action is the covariant derivative, having also a contribution from the gauge vector A_L, which is singular at the origin. Indeed, the constant gauge vector (5.5) is pure gauge, in accordance with (5.7),

$$
A_L = \frac{i}{g} G^{-1}\, dG, \tag{5.10}
$$

with

$$
G = e^{i\phi\tau^3} e^{i(\pi-\theta)\tau^1}\,. \tag{5.11}
$$

Therefore, we can move back to the gauge where $A_L = 0$, and then the second solution (5.9) looks as

$$
c = G(\epsilon\rho\,\tau^3)G^{-1} = \epsilon\left(y\tau^1 + x\tau^2 - z\tau^3\right), \tag{5.12}
$$

which is perfectly regular, where we have changed from the polar coordinates ρ, θ, ϕ to the cartesian ones x, y, z.

We now wish to look at the limit $\rho \to \infty$. There, we can use the singular solution (3.5), (3.6), (3.7) having $a(\rho) = 0$. Equation (5.4) becomes

$$
c_3''(\rho) + \left(2 + \frac{1}{2}\rho^{-1}\right) c_3'(\rho) - \frac{1}{2}\rho^{-2} c_3(\rho) + O(\epsilon^2) = 0\,, \tag{5.13}
$$

two of whose independent solutions are

$$
c_3(\rho) = \epsilon\left(e^{-2\rho}\rho^{-1/2}\right) + O(\epsilon^2)\,, \tag{5.14}
$$

$$
c_3(\rho) = \epsilon\left(1 - \frac{\sqrt{2\pi}}{4} e^{-2\rho}\rho^{-1/2}\,\mathrm{erfi}\left(\sqrt{2\rho}\right)\right) + O(\epsilon^2)\,, \tag{5.15}
$$

where erfi is an imaginary version of the error function, $\mathrm{erfi}(z) = \mathrm{erf}(iz)/i$, obeying $\mathrm{erfi}'(z) = (2/\sqrt{\pi})\exp(+z^2)$. The first solution is exponentially small at $\rho \to \infty$, and is therefore normalizable, while the second behaves as $1 - (1/4)\rho^{-1} + O(\rho^{-2})$ and is non-normalizable. The second solution corresponds to a perturbation of the lagrangian by the appropriate operator $\epsilon\mathcal{O}$, while the first corresponds to a vacuum expectation value (VEV) of the operator. However, we do not expect to have a physically acceptable solution corresponding to a VEV, as the dual field theory does not have a moduli space. Indeed, we will see that the full solution corresponding to having only a VEV is singular at the origin and

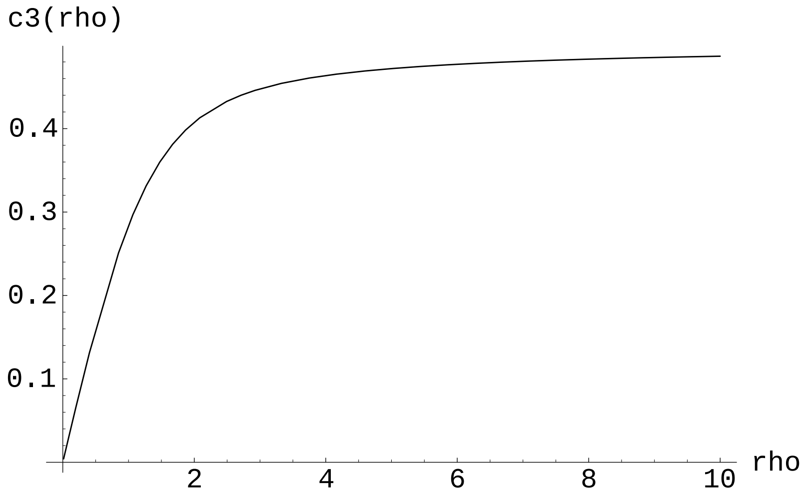

Figure 1: The graph of $c_3(\rho)/\epsilon$ as given in (5.18).

therefore unacceptable. Global regularity of the full solution, or in other words the boundary condition we have described at $\rho = 0$, will determine a particular linear combination of (5.14) and (5.15).

Equation (5.4) can be solved analytically in the whole region $0 \leq \rho < \infty$. Defining

$$q(\rho) = \sqrt{4\rho \sinh(4\rho) - \cosh(4\rho) - 8\rho^2 + 1}\,, \tag{5.16}$$

we have as two independent solutions

$$c_3(\rho) = \epsilon \left(\frac{1}{q(\rho)}\right) + O(\epsilon^2)\,, \tag{5.17}$$

$$c_3(\rho) = \epsilon \left(\frac{\int_0^\rho q(s)\, ds}{q(\rho)}\right) + O(\epsilon^2)\,. \tag{5.18}$$

For small values of ρ we have $q(\rho) = 4\sqrt{2}\,\rho^2 + O(\rho^4)$. Therefore, the first solution behaves at $\rho \to 0$ as ρ^{-2} and is unacceptable, while the second behaves as $(1/3)\rho + O(\rho^3)$ so it is the one we're looking for.

As $\rho \to \infty$, we have

$$q(\rho) = \sqrt{2\rho - \frac{1}{2}} \cdot e^{2\rho} \cdot \left(1 + O(\rho^2 e^{-4\rho})\right). \tag{5.19}$$

If we substitute $q(\rho) \approx \sqrt{2\rho}\, e^{2\rho}$ into (5.18), we get exactly half of (5.15), while clearly the same substitution in (5.17) gives $1/\sqrt{2}$ of (5.14). So, we see again that the solution including the full smoothed background behaves at infinity like the solution including the singular background, with a "lagging" of ρ by $1/4$, up to exponentially small corrections.

To recapitulate, we found three modes of small perturbations around the Maldacena-Nuñez solution: only one (5.18) is a smooth, physically acceptable radial scalar mode $c_3(\rho)$. The second mode (5.17) is singular in the IR, and the third, described by (5.1) with an appropriate solution for $c_3(\rho)$, involves a singular gauge vector A_R.

6. The ten-dimensional solution

The seven-dimensional solution can be lifted up to ten dimensions using the expressions in [22]. In that paper, the three dimensional sphere S^3 used for the lifting is parameterized by μ^a, where $a = 0, 1, 2, 3$ is an SO(4) vector index, and

$$\mu^a \mu^a = 1 \,. \tag{6.1}$$

The covariant derivative of μ^a is defined as

$$\mathcal{D}\mu^a = d\mu^a + \hat{g} A^{ab}_{(1)} \mu^b \,. \tag{6.2}$$

Maldacena and Nuñez [15] employ the identification of S^3 with the group manifold of SU(2), and use instead three Euler angles, which we will denote by $\tilde{\psi}, \tilde{\theta}, \tilde{\phi}$. Their solution can be written in terms of the three one-forms w^l defined by

$$\begin{aligned} w^1 + i w^2 &= e^{-i\tilde{\psi}}(d\tilde{\theta} + i \sin\tilde{\theta} d\tilde{\phi}) \,, \\ w^3 &= d\tilde{\psi} + \cos\tilde{\theta} d\tilde{\phi} \,. \end{aligned} \tag{6.3}$$

The w^l have, in our conventions, an $\mathrm{SU}(2)_L$ index $l = 1, 2, 3$, and are invariant under $\mathrm{SU}(2)_R$.

The translation between these two parameterizations is given by

$$\mu^0 = + \cos\left(\frac{\tilde{\psi} + \tilde{\phi}}{2}\right) \cos\left(\frac{\tilde{\theta}}{2}\right) , \tag{6.4}$$

$$\mu^1 = - \cos\left(\frac{\tilde{\psi} - \tilde{\phi}}{2}\right) \sin\left(\frac{\tilde{\theta}}{2}\right) , \tag{6.5}$$

$$\mu^2 = + \sin\left(\frac{\tilde{\psi} - \tilde{\phi}}{2}\right) \sin\left(\frac{\tilde{\theta}}{2}\right) , \tag{6.6}$$

$$\mu^3 = - \sin\left(\frac{\tilde{\psi} + \tilde{\phi}}{2}\right) \cos\left(\frac{\tilde{\theta}}{2}\right) , \tag{6.7}$$

and we have

$$w^l = \frac{4}{i} (\alpha^l_L)_{ab} \, \mu^a \, d\mu^b \,. \tag{6.8}$$

Those one-forms will appear in the formulae through the combinations

$$\widehat{w}^l \equiv w^l - g A^l_L \,. \tag{6.9}$$

We also have to remember the relation between g and $\hat{g}$ explained in section 2. Because in our solution the $\mathrm{SU}(2)_R$ gauge fields vanish, we find the simple relation

$$\mathcal{D}\mu^a = i \, (\alpha^l_L)_{ab} \, \mu^b \widehat{w}^l \,. \tag{6.10}$$

Using the anti-commutators (A.4) from appendix A, this gives, in particular,

$$\mathcal{D}\mu^a \mathcal{D}\mu^a = \frac{1}{4} \widehat{w}^l \widehat{w}^l \,. \tag{6.11}$$

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The radial solution of the scalar equation which we found in the last section is $c_3(\rho)$ which, without loss of generality, can be taken to be the field $c_{33}(\rho)$. The first order expression in ϵ for the scalars we need to maintain is then, by (2.3), (2.4),

$$\tilde{T}_{ab} = \delta_{ab} + c_3 (X_{33})_{ab} + O(\epsilon^2)\,. \tag{6.12}$$

The lifting up expressions of [22] involve, among other things, the combination $\tilde{T}_{ab}\mu^a\mu^b$. Using the explicit form of X_{33} from appendix A, and (6.4-6.7), we get

$$\begin{aligned} \tilde{T}_{ab}\mu^a\mu^b &= 1 + c_3\left((\mu^0)^2 - (\mu^1)^2 - (\mu^2)^2 + (\mu^3)^2\right) + O(\epsilon^2) \\ &= 1 + c_3\cos\tilde{\theta} + O(\epsilon^2)\,. \end{aligned} \tag{6.13}$$

Another expression which appears in the formulae is

$$\begin{aligned} Z &\equiv (X_{33})_{ab}\mathcal{D}\mu^a\mathcal{D}\mu^b = (\mathcal{D}\mu^0)^2 - (\mathcal{D}\mu^1)^2 - (\mathcal{D}\mu^2)^2 + (\mathcal{D}\mu^3)^2 \\ &= \frac{1}{4}\left(2\sin\tilde{\theta}\,(\sin\tilde{\psi}\,\widehat{w}^1 + \cos\tilde{\psi}\,\widehat{w}^2)\,\widehat{w}^3 - \cos\tilde{\theta}\,((\widehat{w}^1)^2 + (\widehat{w}^2)^2 - (\widehat{w}^3)^2)\right). \end{aligned} \tag{6.14}$$

There is an important point to notice here. In the asymptotic region $\rho \to \infty$, corresponding to the UV, the seven-dimensional solution resembles the singular one, having only an A_L^3 gauge field. Therefore we may take $\widehat{w}^l = w^l$ for $l = 1, 2$ in that region, and it is easily seen that Z involves the coordinate $\tilde{\psi}$ only through $d\tilde{\psi}$. For example,

$$\sin\tilde{\psi}\,w^1 + \cos\tilde{\psi}\,w^2 = \sin\tilde{\theta}\,d\tilde{\phi}\,. \tag{6.15}$$

In other words, Z is asymptotically invariant under a constant shift in $\tilde{\psi}$.

Other expressions for various differential forms which we will find useful are

$$\begin{aligned} \Xi_2 &\equiv \sin\tilde{\theta}\,(\sin\tilde{\psi}\,\widehat{w}^1 + \cos\tilde{\psi}\,\widehat{w}^2)\wedge\widehat{w}^3\,, \\ \Psi_3 &\equiv -(\sin\tilde{\psi}\,gF_L^1 + \cos\tilde{\psi}\,gF_L^2)\wedge\widehat{w}^3 + gF_L^3\wedge(\sin\tilde{\psi}\,\widehat{w}^1 + \cos\tilde{\psi}\,\widehat{w}^2)\,, \\ \Pi_3 &\equiv \sin\tilde{\theta}\,(\sin\tilde{\psi}\,gA_L^1 + \cos\tilde{\psi}\,gA_L^2)\wedge\widehat{w}^1\wedge\widehat{w}^2 + \\ &\quad + \cos\tilde{\theta}\,(gA_L^2\wedge\widehat{w}^1 - gA_L^1\wedge\widehat{w}^2)\wedge\widehat{w}^3\,. \end{aligned} \tag{6.16}$$

Those expressions are also asymptotically invariant under a constant shift in $\tilde{\psi}$ as, by (3.14), (3.19), (3.20), (3.21), A_L^1, A_L^2 and also F_L^1, F_L^2 are asymptotically zero.

We are now in the position to write explicitly the ten-dimensional Einstein frame metric, the dilaton, and the Neveu-Schwarz three-form. We find respectively

$$ds_{10}^2 = e^{-5y/16}\left[\left(1 + \frac{1}{4}c_3\cos\tilde{\theta}\right)ds_{7,\mathrm{st}}^2 + \frac{2}{g^2}\left(1 - \frac{3}{4}c_3\cos\tilde{\theta}\right)\left(\frac{1}{4}\widehat{w}^l\widehat{w}^l - c_3 Z\right)\right] + O(\epsilon^2)\,, \tag{6.17}$$

$$\begin{aligned} \varphi &= \frac{5}{8}y - \frac{1}{2}c_3\cos\tilde{\theta} + O(\epsilon^2) \\ &= \varphi_0 + \frac{1}{2}\log\left(\frac{2e^{h(\rho)}}{\sinh(2\rho)}\right) - \frac{1}{2}c_3\cos\tilde{\theta} + O(\epsilon^2)\,, \end{aligned} \tag{6.18}$$

$$\begin{aligned} H^{\mathrm{NS}} &= \frac{2}{g^2}\left[-\frac{1}{4}(1 - 2c_3\cos\tilde{\theta})\widehat{w}^1\wedge\widehat{w}^2\wedge\widehat{w}^3 + \frac{1}{4}gF_L^l\wedge\widehat{w}^l + \frac{1}{4}c_3\sin\tilde{\theta}\,\Psi_3 - \right. \\ &\quad \left. - \frac{1}{4}\,c_3\,\Pi_3 - \frac{1}{4\sqrt{2}}\,\Xi_2\wedge dc_3\right] + O(\epsilon^2)\,. \end{aligned} \tag{6.19}$$

In those formulae we have used the Einstein summation convention, and in our radial case we have $dc_3 = c_3'(\rho)\, d\rho$. Note that the combination $c_3 \cos\tilde{\theta}$, which appears above in the expressions for fields which are $\mathrm{SU}(2)_R$-invariant, is indeed covariant under $\mathrm{SU}(2)_R$.

Those formulae reduce to the solution of [15] when the scalar $c_3(\rho)$ is turned off. In our formulation, however, an explicit factor of g accompanies each A_L^l or F_L^l, as in (6.9), because of the prefactor $1/g$ in the ansatz (3.14). The additional terms in our solution are proportional to the scalar $c_3(\rho)$, given by (5.18), and thus to the small parameter ϵ in which we expand.

It is interesting to note that in the string frame, the seven-dimensional part of the ten-dimensional metric does not contain a contribution linear in ϵ,

$$ds^2_{10,\mathrm{st}} = ds^2_{7,\mathrm{st}} + \frac{2}{g^2}(1 - c_3\cos\tilde{\theta})\left(\frac{1}{4}\widehat{w}^l\widehat{w}^l - c_3 Z\right) + O(\epsilon^2)\,. \tag{6.20}$$

7. The S-dual background and the string tension

In order to find the tension of the Wilson line in the field theory it will be convenient to S-dualize the background, or to wrap D5 branes on the two-sphere instead of NS5 branes. The string frame metric S-dual to (6.20) is

$$ds^2_{10,\mathrm{st},D} = e^{\varphi_D} ds^2_{10,\mathrm{st}}\,, \tag{7.1}$$

where the S-dual dilaton is given by

$$\varphi_D = \varphi_{D,0} - \frac{1}{2}\log\left(\frac{2e^{h(\rho)}}{\sinh(2\rho)}\right) + \frac{1}{2}c_3\cos\tilde{\theta} + O(\epsilon^2)\,, \tag{7.2}$$

and the Neveu-Schwarz field H^{NS} turns into the S-dual Ramond-Ramond field H^{RR}.

We may probe the YM theory by heavy external quarks. The fluxtube between a quark anti-quark pair is described in the S-dual gravitational dual by a fundamental string, lying on a geodesic of the dual supergravity background, and asymptoting to $\rho = \infty$ at both ends [26, 27]. The confining potential is given by the renormalized mass of such a string. For large separations, the string will tend to be stretched where the four-dimensional metric is minimal, the string tension will be given in terms of the metric at that minimum, and the corrections to the linear potential are very small [28]. In our case, the minimum is obtained where φ_D attains the minimal value. Before the perturbation, φ_D had a quadratic minimum at the origin $\rho = 0$. The perturbation by c_3 adds a function (5.18) which is linear in ρ near the origin and is proportional to the small parameter ϵ times $\cos\tilde{\theta}$. Taking $\epsilon > 0$ the minimum will be attained for $\cos\tilde{\theta} = -1$, that is at the south pole $\tilde{\theta} = \pi$ of the three-sphere, and for $\rho_{\min} = \frac{3}{16}\epsilon$. However, the value of the metric at the minimum arising from this shift is only corrected at quadratic order in ϵ, and it might be affected also by terms in φ_D which are quadratic in ϵ. All we can say is, therefore, that $\varphi_{D,\min} \equiv \varphi_D(\rho_{\min}) = \varphi_{D,0} + O(\epsilon^2)$. The string tension $T_{\mathrm{st}} = e^{\varphi_{D,\min}}/2\pi\alpha'$ may then also be corrected relative to the unperturbed value, with a correction which is quadratic in the strength of the (small) perturbation.

Other properties of the dual gauge dynamics like the Lüscher term, the broadening of the flux tube, stringy corrections to the intercept, 't Hooft loops, baryonic configurations, the gaugino condensate and the domain wall tensions will behave as in the Maldacena-Nuñez case [15, 29, 30]. The U(1) R-symmetry corresponds in the ten–dimensional picture to a constant shift of the $\tilde{\psi}$ coordinate [15]. As shown in the previous section, this is a symmetry of the asymptotic form of our solution for large ρ. The breaking of this symmetry by worldsheet instantons will also remain as in [15], except for a small change in the shape of the dominant worldsheet configurations.

8. Supersymmetry and its breaking

In this section we exhibit explicitly the supersymmetry of the Maldacena-Nuñez solution, and we show that it is broken by the deformation. It is sufficient for us to work in the singular solution, which is simpler, because we break supersymmetry in the UV, where the two solutions are similar. Supersymmetry acts locally, so the UV behaviour should not be affected by the smoothing in the IR region. We work in ten dimensions, in the context of type IIB supergravity, although the solution is contained in the sector of type I fields. The four-dimensional Minkowski space $x^\mu, \mu = 0, 1, 2, 3$ plays no role in the considerations, and therefore we may work solely with the six coordinates $\rho, \theta, \phi, \tilde{\theta}, \tilde{\phi}, \tilde{\psi}$. We choose an eight-dimensional Majorana representation for the flat Euclidean SO(6) Clifford algebra, that is, purely imaginary gamma matrices $\gamma^i, i = 1, \ldots, 6$ satisfying $\{\gamma^i, \gamma^j\} = 2\delta^{ij}$. The gamma matrices for our six-dimensional curved space will be denoted by $\Gamma^M, M = 1, \ldots, 6$, where $\Gamma^M = e^{iM}\gamma^i$, with e^{iM} being the vielbein, $e^i_M e^i_N = g_{MN}$.

The background is bosonic, and therefore the supersymmetry transformations of the bosons vanish, and those of the fermions involve only the bosonic fields. Remember also that our background contains no five-form field. In our conventions, the supersymmetry transformations with a spinor parameter η for the dilatino and gravitino read, respectively,

$$\delta\lambda = \frac{i}{2}\Gamma^M \partial_M \varphi\, \eta^* - \frac{i}{24} e^{-\varphi/2} \Gamma^{MNP} H_{MNP}\, \eta\,, \tag{8.1}$$

$$\delta\psi_M = \mathcal{D}_M \eta + \frac{1}{96} e^{-\varphi/2} \left(\Gamma_M{}^{NPQ} H_{NPQ} - 9\Gamma^{NP} H_{MNP} \right) \eta^*\,. \tag{8.2}$$

The covariant derivative of the spinor is $\mathcal{D}_M = \partial_M + \frac{i}{2}\omega_M^{ij}\Sigma^{ij}$, where ω_M^{ij} is the spin connection, and $\Sigma^{ij} = -\frac{i}{4}[\gamma^i, \gamma^j]$ are the generators of rotations in the spinor representation. The multiple index gamma matrices are defined as antisymmetrized products of unit weight, that is, for example, $\Gamma^{MN} = \frac{1}{2!}(\Gamma^M\Gamma^N - \Gamma^N\Gamma^M)$. Since our gamma matrices are imaginary, the supersymmetry transformations (8.1), (8.2) are real operators, acting both on real and on imaginary spinors. Hence the equations for a complex spinor η decompose into equations for its real part and for its imaginary part, and we can work separately with the two cases.

The unperturbed singular solution is rather inelaborate. The dilaton and Neveu-Schwarz three-form are

$$\varphi = \varphi_0 - \rho + \frac{1}{4}\log\rho\,, \tag{8.3}$$

$$H^{\rm NS} = \frac{2}{g^2}\left(-\frac{1}{4}\right)\Bigg[\sin\tilde\theta(d\tilde\theta\wedge d\tilde\phi\wedge d\tilde\psi - \cos\theta\, d\phi\wedge d\tilde\theta\wedge d\tilde\phi) + $$
$$+ \sin\theta(d\theta\wedge d\phi\wedge d\tilde\psi + \cos\tilde\theta\, d\theta\wedge d\phi\wedge d\tilde\phi)\Bigg]. \tag{8.4}$$

This considerably simplifies the supersymmetry equations. Choosing η real, $\eta^* = \eta$, and writing $\delta\lambda = M^{(0)}\eta$, we find that the matrix $M^{(0)}$ has a two-dimensional kernel, $\mathrm{Ker}\, M^{(0)} = \mathrm{Span}\{\zeta_1, \zeta_2\}$ (see appendix C for some explicit details). Specializing to this subspace, i.e. writing

$$\eta = B_1\zeta_1 + B_2\zeta_2\,, \tag{8.5}$$

the equations for the gravitino, $\delta\psi_M = 0$, are satisfied for $M = \theta, \phi, \tilde\theta, \tilde\phi$ with vanishing ordinary derivative, so the solution is consistent with no dependence of η on those coordinates. From the equation $\delta\psi_\rho = 0$ we get that

$$\begin{aligned} \partial_\rho B_1 - \frac{1}{8}\left(1 - \frac{1}{4\rho}\right) B_1 &= 0\,, \\ \partial_\rho B_2 - \frac{1}{8}\left(1 - \frac{1}{4\rho}\right) B_2 &= 0\,, \end{aligned} \tag{8.6}$$

from which we extract the radial dependence of the spinor: both B_1 and B_2 are proportional to $\rho^{-1/32}e^{\rho/8}$.

Finally, from the equation $\delta\psi_{\tilde\psi} = 0$ we get

$$\begin{aligned} \partial_{\tilde\psi} B_1 + \frac{1}{2}B_2 &= 0\,, \\ \partial_{\tilde\psi} B_2 - \frac{1}{2}B_1 &= 0\,. \end{aligned} \tag{8.7}$$

The solution looks like $B_1 \sim \cos(\tilde\psi/2)\,,\, B_2 \sim \sin(\tilde\psi/2)$ up to an overall factor and a constant shift in $\tilde\psi$. We find that the spinors have the appropriate charge under the U(1) R-symmetry taking $\tilde\psi \to \tilde\psi + \delta\tilde\psi$, and that they acquire a phase of -1 when $\tilde\psi \to \tilde\psi + 2\pi$. In particular [15], the periodicity of $\tilde\psi$ is $\tilde\psi \equiv \tilde\psi + 4\pi$.

All in all, we get that the remaining supersymmetry is generated by the spinors

$$\eta = E\rho^{-1/32}e^{\rho/8}\left(\cos\frac{\tilde\psi - \tilde\psi_0}{2}\,\zeta_1 + \sin\frac{\tilde\psi - \tilde\psi_0}{2}\,\zeta_2\right), \tag{8.8}$$

where E and $\tilde\psi_0$ are arbitrary constants. Choosing η to be imaginary, $\eta^* = -\eta$, we still get a two-dimensional solution for the dilatino equation, but this solution is not consistent with the gravitino equations. Therefore we are left with one eighth of type IIB supersymmetry, or one fourth of type I in ten dimensions, which corresponds to $\mathcal{N} = 1$ in four dimensions.

Next, we wish to show that the perturbation by the radial scalar (in the seven-dimensional language) indeed totally breaks supersymmetry. To this end it is enough to show that the dilatino equation cannot continue to be satisfied in the presence of the perturbing field c_3. The supersymmetry transformations of the dilatino and gravitino are modified, and the spinor η also might change, $\eta = \eta^{(0)} + \epsilon\eta^{(1)} + O(\epsilon^2)$, where $\eta^{(0)}$ is the real solution (8.8) to the unperturbed equation, $M^{(0)}\eta^{(0)} = 0$. If $\eta^{(1)}$ has an imaginary part, then the imaginary parts of the $O(\epsilon)$ supersymmetry equations are identical to the original equations on imaginary spinors, which we found to have no solutions. Therefore we can assume that $\eta^{(1)}$ is also real.

The supersymmetry transformation of the dilatino is changed to $\delta\lambda = M\eta$ where $M = M^{(0)} + \epsilon M^{(1)} + O(\epsilon^2)$. Positing that supersymmetry might be conserved, we must have

$$\begin{aligned} 0 = \delta\lambda = M\eta &= \left(M^{(0)} + \epsilon M^{(1)}\right)(\eta^{(0)} + \epsilon\eta^{(1)}) + O(\epsilon^2) \\ &= \epsilon\left(M^{(1)}\eta^{(0)} + M^{(0)}\eta^{(1)}\right) + O(\epsilon^2)\,, \end{aligned} \tag{8.9}$$

or

$$M^{(0)}\eta^{(1)} = -M^{(1)}\eta^{(0)}\,, \tag{8.10}$$

which can be stated as $M^{(1)}\eta^{(0)} \in \operatorname{Image} M^{(0)}$, or finally as $M^{(1)}\eta^{(0)} \perp \left(\operatorname{Image} M^{(0)}\right)^{\perp}$. Checking this explicitly (see appendix C) yields the condition

$$c_3'(\rho) + \frac{1+4\rho}{2\rho}c_3(\rho) = 0\,. \tag{8.11}$$

Remember that this is a necessary condition for having some remaining supersymmetry, coming only from the dilatino equation.

Taking the non-normalizable radial scalar mode (5.15) as the perturbation of the singular background, we see that (8.11) is not satisfied. Therefore, this perturbation, which is dual to perturbing the lagrangian in the field theory, breaks supersymmetry.

It is interesting to note that (8.11) is satisfied for the normalizable radial scalar mode (5.14) of the same background. This might suggest that supersymmetry is conserved for such a deformation of the solution (although the gravitino equations should also be checked in order to verify this). This is natural from the dual viewpoint of the field theory, where such a mode corresponds to having a VEV of the operator $\mathcal{O}$, which does not break supersymmetry. However, the physical relevance of such a mode is not clear since the true, smooth, solution has no normalizable mode, and the field theory has no moduli space.

9. Summary and discussion

We began by expanding the seven-dimensional $\mathrm{SO}(4) \equiv \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ gauged supergravity lagrangian (2.18) up to second order in the $\mathbf{9} = (\mathbf{3},\mathbf{3})$ scalar fields c_{lr}. We reviewed the singular solution (3.5), (3.6), (3.7) and the smooth Chamseddine-Volkov solution (3.17), (3.18), (3.19) of that lagrangian and studied their symmetries. We then took the scalars to be proportional to the small parameter ϵ, and wrote their linearized (that is, $O(\epsilon)$) equations of motion. Those equations split into three identical copies, indexed by $r = 1, 2, 3$, of decoupled equations, and any superposition of such solutions c_{lr} can be brought to the form c_{l3} by a suitable $\mathrm{SU}(2)_R$ rotation. We therefore wrote those equations (4.2), (4.3) in terms of a single $\mathrm{SU}(2)_L$ triplet of scalars c_l. Working around the aforementioned singular and smooth solutions, and keeping their symmetries, we found that the perturbation must be a radial field $c_3(\rho)$ obeying the equation of motion (5.4).

Around the singular solution we found three independent modes. The first (5.14) is normalizable and corresponds to a VEV of the operator $\mathcal{O} = \mathrm{tr}\left(\Phi_1^2 - \Phi_2^2 - \Phi_3^2 + \Phi_4^2\right)$ in the dual six-dimensional field theory. The second (5.15) is non-normalizable and corresponds to a perturbation by that operator, and the third mode includes a singular gauge field A_R given by (5.1). Around the physical, smooth solution, we found that only the second of these modes (5.18), corresponding to the perturbation by $\epsilon\mathcal{O}$, is smooth and physically acceptable, the other two being singular in the IR. Indeed, the dual field theory has no moduli space and therefore no VEV is possible. Then, we raised the seven-dimensional solution to a ten-dimensional one (6.17), (6.18), (6.19). This is a stable solution as it is dual to a field theory exhibiting a mass gap.

The most important remaining issue is to find the solution to order ϵ^2. This involves finding the back-reaction of the perturbation $c_3(\rho)$ on the fields of the Chamseddine-Volkov background in seven dimensions, and raising it again to ten dimensions. In order to find the back-reaction we should retain in the lagrangian (2.18) all the $O(\epsilon^2)$ terms. Working with a solution and a perturbation which both do not involve the $\mathrm{SU}(2)_R$ gauge vectors A_R^r to first order in ϵ, as we must for the physical perturbation, those gauge vectors can be neglected also in the back-reaction, because they do not contribute to the lagrangian to order ϵ^2. In particular

$$\mathcal{D}_\mu c_l = \partial_\mu c_l + g\epsilon_{lmn} A^m_{L\mu} c_n \, , \tag{9.1}$$

and the relevant lagrangian takes the form

$$L_7 = R - \frac{5}{16}\partial_\mu y \partial^\mu y - \mathcal{D}_\mu c_l \mathcal{D}^\mu c_l - \frac{1}{4}e^{-y/2}(F_L^l F_L^l + 2F_L^l F_L^m c_l c_m) + 2g^2 e^{y/2} + O(\epsilon^3) \, . \tag{9.2}$$

The equations of motions arising from this lagrangian at second order in ϵ are cumbersome coupled equations, but can be solved in principle.

The back-reaction computation is necessary for computing the energy of the vacuum (such a computation was carried out in a different context in [31]). Since supersymmetry is broken at order ϵ, we expect to have a non-zero vacuum energy of order ϵ^2; note that since the deformation is in the **3** representation of $\mathrm{SU}(2)_R$, all singlets of $\mathrm{SU}(2)_R$ (such as the vacuum energy) must depend at least quadratically on ϵ. In particular, we saw this in the computation of the string tension. It would be interesting to complete this calculation, as well as to find the corrections to the glueball masses and to the domain wall tensions (the domain walls are no longer BPS in the deformed solution).

In section 8, we explicitly demonstrated that supersymmetry is broken in ten dimensions by the small perturbation of the background. It would be nice to be able to check this directly in seven dimensions. Such a computation would be somewhat more direct and simple, and would presumably allow to deal in a feasible manner even with the smooth background. The supersymmetry transformations for the seven-dimensional SO(4) gauged supergravity including the nonet of scalars can be extracted from [32].

Even though the perturbation we described breaks supersymmetry, it does not break the U(1) R-symmetry, so many of the qualitative features of the solution of [15] persist. In particular, through the anomalous and spontaneous breaking of this symmetry we are still left with N equivalent vacua, which are permuted by the action of $\mathbb{Z}_N$. These vacua, in the language of the YM theory, differ only in the phase of the gaugino bilinear condensate. The linearized breaking of supersymmetry involves only the scalars Φ_a, and it does not couple directly to the low-energy YM theory. It would be interesting to analyze other supersymmetry-breaking deformations which would not sit in the same direction of the group space as the twisting of the normal bundle, which in our case corresponds to the adjoint index $l = 3$ of $\mathrm{SU}(2)_L$. Such deformations would explicitly break the U(1) R-symmetry, and we showed that they would not be purely radial in seven dimensions, so they would be more difficult to deal with.

A different supergravity background, which is also related in some limit to four-dimensional $\mathcal{N} = 1$ pure SYM theory, was found in [33]. It would be interesting to find supersymmetry breaking deformations, similar to the one we discuss here, also for that background. In that case the UV theory is less well-understood, so it is not obvious à priori what supersymmetry-breaking deformations can be performed.

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A. Some SO(4) conventions

We define a basis for the SO(4) Lie algebra :

$$\alpha_L^1 = -\frac{i}{2}\begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}, \quad \alpha_L^2 = -\frac{i}{2}\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad \alpha_L^3 = -\frac{i}{2}\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \tag{A.1}$$

and

$$\alpha_R^1 = -\frac{i}{2}\begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}, \quad \alpha_R^2 = -\frac{i}{2}\begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad \alpha_R^3 = -\frac{i}{2}\begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}. \tag{A.2}$$

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These conventions are closely related, but not identical, to those of [19]. The commutators of those generators are

$$\left[\alpha_L^l, \alpha_L^m\right] = i\epsilon_{lmn}\alpha_L^n \tag{A.3}$$

and similarly for the α_R^r, as needed for the SU(2) algebra. The anti-commutators are

$$\left\{\alpha_L^l, \alpha_L^m\right\} = \frac{1}{2}\delta_{lm} \tag{A.4}$$

and similarly for the α_R^r.

Now we define a basis for the $(\mathbf{3},\mathbf{3})$ representation (acting on two SO(4) vectors):

$$X_{11} = \begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad X_{12} = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & +1 & 0 \\ 0 & +1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \quad X_{13} = \begin{pmatrix} 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & +1 \\ +1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \end{pmatrix},$$
$$X_{21} = \begin{pmatrix} 0 & 0 & 0 & +1 \\ 0 & 0 & +1 & 0 \\ 0 & +1 & 0 & 0 \\ +1 & 0 & 0 & 0 \end{pmatrix}, \quad X_{22} = \begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad X_{23} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1 \\ 0 & 0 & +1 & 0 \end{pmatrix},$$
$$X_{31} = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & +1 \\ -1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \end{pmatrix}, \quad X_{32} = \begin{pmatrix} 0 & +1 & 0 & 0 \\ +1 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1 \\ 0 & 0 & +1 & 0 \end{pmatrix}, \quad X_{33} = \begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & +1 \end{pmatrix}. \tag{A.5}$$

The matrix X_{33}, say, can be determined by demanding of the left lowering operator $a_L^3 \equiv \alpha_L^1 - i\alpha_L^2$ that $a_L^{3\,\dagger}\, X_{33}\, a_L^3 = 0$, and similarly with a_R^3. The X_{lr} can be seen to transform properly under the rotations generated by α_L^l and α_R^r. With those conventions, we have $X_{lr} = -4\,\alpha_L^l\,\alpha_R^r$.

B. Some seven-dimensional supergravity calculations

In this appendix we translate the various terms in the lagrangian (2.12) into the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ language. We work to quadratic order in the fields c_{lr}, but we will comment on the way to get higher order expressions.

In the kinetic term of the scalars in the **9** representation, in order to maintain only quadratic expressions in c, it is sufficient to substitute

$$\tilde{T}_{ab} = \delta_{ab} + O(c) \tag{B.1}$$

for $\tilde{T}$ (but not for its covariant derivative), since the derivatives are at least of first order in c. Thus, this term takes the form

$$-\frac{1}{4}\mathcal{D}_\mu \tilde{T}_{ab}\mathcal{D}^\mu \tilde{T}_{ba} + -O(c^3) = \frac{1}{4}\operatorname{tr}\mathcal{D}_\mu \tilde{T}\mathcal{D}^\mu - \tilde{T} + O(c^3)\,. \tag{B.2}$$

We now use the formula for the derivative of a matrix exponent, used in the proof of the Baker-Campbell-Hausdorff formula,

$$\left(e^Q\right)' = e^Q \left(Q' - \frac{1}{2!}[Q, Q'] + \frac{1}{3!}[Q,[Q,Q']] - \cdots \right) . \tag{B.3}$$

For our purposes, it is sufficient to maintain simply

$$\tilde{T}' = Q' + O(c^2) . \tag{B.4}$$

Using (2.7) we get

$$\mathcal{D}_\mu \tilde{T} = \partial_\mu Q + \hat{g}[A, Q] + O(c^2) . \tag{B.5}$$

Substituting we get that the kinetic term of the c_{lr} fields is

$$-\mathcal{D}_\mu c_{lr} \mathcal{D}^\mu c_{lr} , \tag{B.6}$$

where

$$\mathcal{D}_\mu c_{lr} = \partial_\mu c_{lr} - \sqrt{2}\hat{g}(\epsilon_{lmn} A_L^m c_{nr} + \epsilon_{rst} A_R^s c_{lt}) . \tag{B.7}$$

For the Maxwell term, which can be written as

$$\frac{1}{8} e^{-y/2} \operatorname{tr} F_{\mu\nu} \tilde{T}^{-1} F^{\mu\nu} \tilde{T}^{-1} , \tag{B.8}$$

one needs to substitute the quadratic approximation of $\tilde{T}$,

$$\tilde{T}^{-1} = 1 - Q + \frac{1}{2} Q^2 + O(c^3) . \tag{B.9}$$

We find that this term is given by

$$-\frac{1}{4} e^{-y/2} \left(F_L^l F_L^l + F_R^r F_R^r + 4 F_L^l F_R^r c_{lr} + 2(F_L^l F_L^m c_{lr} c_{mr} + F_R^r F_R^s c_{lr} c_{ls}) \right) + O(c^3) , \tag{B.10}$$

where, for example, $F_L^l F_L^l \equiv F_L^{l\mu\nu} F_{L\mu\nu}^l$.

Finally, the potential term is

$$V = -\frac{1}{2} \hat{g}^2 e^{y/2} (2 \operatorname{tr} \tilde{T}^2 - (\operatorname{tr} \tilde{T})^2) . \tag{B.11}$$

Using the quadratic approximation of $\tilde{T}$ and (2.4) one gets simply that

$$V = 4\hat{g}^2 e^{y/2} + O(c^3) . \tag{B.12}$$

If one were interested in higher orders of c in the potential, one could use the characteristic polynomial of Q and the Vieta identities. This polynomial, $p(\lambda) = \det(\lambda 1 - Q)$, is an $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ invariant of c_{lr}, which is homogeneous of degree four in those fields and in λ. Specifically,

$$p(\lambda) = \lambda^4 - 2I_2 \lambda^2 - 8 I_3 \lambda + 2 I_4 - I_2^2 , \tag{B.13}$$

where the three invariants are

$$\begin{aligned} I_2 &= c_{lr} c_{lr} = \operatorname{tr} cc^T , \\ I_3 &= \frac{1}{6} \epsilon_{lmn} \epsilon_{rst} c_{lr} c_{ms} c_{nt} = \det c , \\ I_4 &= c_{lr} c_{ls} c_{ms} c_{mr} = \operatorname{tr} cc^T cc^T . \end{aligned} \tag{B.14}$$

C. Some six-dimensional supersymmetry calculations

We work with the following representation of the flat SO(6) gamma matrices:

$$
\gamma^1 = i\begin{pmatrix}
0&0&0&0&0&0&0&+1\\
0&0&0&0&0&0&+1&0\\
0&0&0&0&0&-1&0&0\\
0&0&0&0&-1&0&0&0\\
0&0&0&+1&0&0&0&0\\
0&0&+1&0&0&0&0&0\\
0&-1&0&0&0&0&0&0\\
-1&0&0&0&0&0&0&0
\end{pmatrix}, \quad
\gamma^2 = i\begin{pmatrix}
0&0&0&-1&0&0&0&0\\
0&0&-1&0&0&0&0&0\\
0&+1&0&0&0&0&0&0\\
+1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&+1\\
0&0&0&0&0&0&+1&0\\
0&0&0&0&0&-1&0&0\\
0&0&0&0&-1&0&0&0
\end{pmatrix},
$$

$$
\gamma^3 = i\begin{pmatrix}
0&0&0&0&0&0&-1&0\\
0&0&0&0&0&0&0&+1\\
0&0&0&0&-1&0&0&0\\
0&0&0&0&0&+1&0&0\\
0&0&+1&0&0&0&0&0\\
0&0&0&-1&0&0&0&0\\
+1&0&0&0&0&0&0&0\\
0&-1&0&0&0&0&0&0
\end{pmatrix}, \quad
\gamma^4 = i\begin{pmatrix}
0&0&-1&0&0&0&0&0\\
0&0&0&+1&0&0&0&0\\
+1&0&0&0&0&0&0&0\\
0&-1&0&0&0&0&0&0\\
0&0&0&0&0&0&-1&0\\
0&0&0&0&0&0&0&+1\\
0&0&0&0&+1&0&0&0\\
0&0&0&0&0&-1&0&0
\end{pmatrix},
$$

$$
\gamma^5 = i\begin{pmatrix}
0&0&0&0&0&+1&0&0\\
0&0&0&0&+1&0&0&0\\
0&0&0&0&0&0&0&+1\\
0&0&0&0&0&0&+1&0\\
0&-1&0&0&0&0&0&0\\
-1&0&0&0&0&0&0&0\\
0&0&0&-1&0&0&0&0\\
0&0&-1&0&0&0&0&0
\end{pmatrix}, \quad
\gamma^6 = i\begin{pmatrix}
0&+1&0&0&0&0&0&0\\
-1&0&0&0&0&0&0&0\\
0&0&0&+1&0&0&0&0\\
0&0&-1&0&0&0&0&0\\
0&0&0&0&0&+1&0&0\\
0&0&0&0&-1&0&0&0\\
0&0&0&0&0&0&0&+1\\
0&0&0&0&0&0&-1&0
\end{pmatrix}. \tag{C.1}
$$

Then, $M^{(0)}$ defined in section 8 is given by

$$
M^{(0)} = P\begin{pmatrix}
0&0&0&0&1&0&4\rho&-1+4\rho\\
0&0&0&0&0&1&-1+4\rho&4\rho\\
0&0&0&0&-4\rho&1-4\rho&-1&0\\
0&0&0&0&1-4\rho&-4\rho&0&-1\\
1&0&-4\rho&-1+4\rho&0&0&0&0\\
0&1&-1+4\rho&-4\rho&0&0&0&0\\
4\rho&1-4\rho&-1&0&0&0&0&0\\
1-4\rho&4\rho&0&-1&0&0&0&0
\end{pmatrix}, \tag{C.2}
$$

where the prefactor is

$$
P = \frac{1}{8\sqrt{2}}\, g\, e^{\varphi/4}\, \rho^{-1}\,, \tag{C.3}
$$

and for φ we should take the unperturbed value given by (8.3).

The kernel of $M^{(0)}$ is given by $\mathrm{Ker}\, M^{(0)} = \mathrm{Span}\{\zeta_1, \zeta_2\}$ where we may take

$$\begin{aligned} \zeta^1 &= (+1,+1,+1,+1,0,0,0,0)^T\,, \\ \zeta^2 &= (0,0,0,0,+1,-1,-1,+1)^T\,. \end{aligned} \tag{C.4}$$

The orthogonal space to the image of $M^{(0)}$ is given by $\left(\mathrm{Image}\, M^{(0)}\right)^{\perp} = \mathrm{Span}\{\xi_1, \xi_2\}$ where we may take

$$\begin{aligned} \xi^1 &= (0,0,0,0,-1,-1,+1,+1)\,, \\ \xi^2 &= (-1,+1,-1,+1,0,0,0,0)\,. \end{aligned} \tag{C.5}$$

The matrix $M^{(1)}$ is too cumbersome to write explicitly, but we have

$$\begin{aligned} \xi^1 M^{(1)} \zeta^1 &= \xi^2 M^{(1)} \zeta^2 = -4\cos\tilde{\theta}\, P\, \left(2\rho\, c_3'(\rho) + (1+4\rho)\, c_3(\rho)\right), \\ \xi^2 M^{(1)} \zeta^1 &= \xi^1 M^{(1)} \zeta^2 = 0\,, \end{aligned} \tag{C.6}$$

so the necessary condition for supersymmetry to be conserved is given by (8.11).

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